University of International Business and Economics

University of International Business and Economics
International Summer School

MAT 230 Multivariable Calculus (Calculus III)

Term: May 25th – June 25th, 2020
Instructor: Dr. Sergei V. Shabanov
Home Institution: University of Florida, Gainesville, USA
Class Hours: Monday through Thursday, 120 minutes each day (2,400 minutes in total)
Office hours: to be announced
Discussion sessions: each Wednesday, time TBD
Email: shabanov@ufl.edu
Webpage: people.clas.ufl.edu/shabanov
Teaching Assistant: TBD

Total Contact Hours: 66 contact hours (45 minutes each, 48 hours in total)
Credit: 4 units

Course Description:

The course covers the following concepts: vector algebra, lines, planes, curves, and surfaces in space, functions of several variables, multivariable limits and continuity, partial derivatives and differentiation of functions of several variables, extreme values of functions of several variables and the method of Lagrange multipliers, double and triple integrals, change of variables in multiple integrals, line and surface integrals, and applications of differentiation and multiple integration to vector fields (line and surface (flux) integrals of vector fields, fundamental theorem for line integrals, etc.).

Course Goals:

A student who satisfactorily completes this course should be able to:

1. Use the rules of vector algebra to describe lines, planes, and curves in space;
2. Analyze functions of several variables using rules of differentiation;
3. Investigate extreme values of functions of several variables;
4. Evaluate multiple integrals;
5. Evaluate line and surface integrals of vector fields.

Required Text:

S.V. Shabanov, Concepts in Calculus III. Multivariable Calculus,
ISBN 978-1-61610-162-6, Edition of 2019.

A free PDF file of the latest (2019) edition of the textbook will be provided to all students enrolled into the course. Please do not buy a beta version (2011) of the book online. The students are expected to read ALL examples and recommended Study Problems in the textbook.

Prerequisites:

The course is based on Calculus 1 and 2 (or their equivalents). Students are expected to know basic concepts of calculus for functions of a single real variable. Good technical skills in differentiation and integration are necessary. Particular topics of Calculus 2, such as numerical series, power series, planar curves, are not mandatory for the course. However, a basic knowledge of these topics will be very helpful as the course contains higher dimensional versions of them.

Exams:

There will be five one-hour exams (one per each chapter of the course). The exam dates are given in the course schedule below. Each exam contains 8 problems, 5-6 of which are taken directly from the homework assignment for the week prior to the exam or from Examples discussed in the textbook. The other 3-2 problems are conceptually similar to the homework problems. There will be one non-standard extra credit problem in each exam. A good way to prepare for such problems is to read the solutions of the Study Problems in the textbook. All exams are free-response assignments. No credit for plain answers. The logic and technical details of solutions must be given in order to get a credit. No notes, no books, no calculators or any other electronic devices are permitted on the exams. One formula sheet is permitted. Makeups for missed exams are only with a written medical excuse approved by the school administration.

Grading Policy:

In five written assignments, there are total 40 problems. Each problem is worth a point if it is solved correctly. So, the perfect score is 40 for regular problems (and 45 with all extra credit problems). There is a small partial credit for incomplete solutions (a fraction of a point). If P is the total number of points, then the course score G=100*(P/40), that is, G=100 if P=40 and G=50 if P=20, etc.

Grading Policy:

If G is the course score as defined above, then the grade thresholds are

A	90 and above	**C+**	65-69
A-	85-89	**C**	60-64
B+	80-84	**C-**	55-59
B	75-79	**D**	50-54
B-	70-74	**F**	below 50

It should be noted that in many US colleges **C-** is not a passing grade if the course is required for a major.

General expectations

Students are expected to:

- *Do the homework regularly, even though the homework is not to be turned in. Remember that to get points toward your grade, every week you have to solve 5-6 problems randomly chosen from homework problems and 3-2 problems very similar to those in the homework or Examples in the textbook. So, read Examples and Study Problems in the textbook while doing the homework and review your solutions before exams.*
- *Review class notes and, if necessary, read the corresponding sections in the textbook, clarify questions about basic concepts of the course during office hours.*
- *Attend all classes and be responsible for all material covered in class and otherwise assigned. Any unexcused absence may impact a student's grade.*
- *Refrain from texting, phoning or engaging in computer activities unrelated to class during class. Students who do not do this will be asked to leave the class*
- *Participate in class discussions and complete required written work on time. While class participation is welcome, even required, you are expected to refrain from private conversations during the class period.*

Attendance policy：

Summer school is very intense and to be successful, students need to attend *every class*. Occasionally, due to illness or other unavoidable circumstance, a student may need to miss a class. A medical certificate is required to be excused. Any absence may impact on the student's grade. Arriving late or leaving early will count as a partial absence. If a student is missing less than a point for a better grade, the better grade will be given, provided the student had no unexcused absences during the course.

Academic honesty：

Students are expected to maintain high standards of academic honesty. Specifically, no notes, no electronic devices, no books are permitted on Exams. One formula sheet, *written by yourself,* on a piece of paper of a standard format (e.g., A4) is allowed on Exams (no torn pages from a book or similar). Admission to Exams is only with a picture ID. Zero tolerance to any kind of cheating (e.g., copying solutions from classmates, use of unauthorized materials or devices). Failure to abide by these rules will result in a zero score on the examination, or even failure in the course.

Course schedule：

The planned schedule given below may be modified to suit the interests or abilities of the enrolled students or to take advantage of special opportunities or events that may arise during the term. There will be a discussion session every Wednesday evening to discuss homework assignments. Students are also encouraged to organize a chat group supervised by TAs to get help on solving homework problems (to avoid multiple discussions of the same problem). Answers to the homework problems are given at the end of each chapter of the textbook.

WEEK ONE (May 25 – May 28): Vector algebra, lines and planes in space

Mon: Sections 1-3. Euclidean geometry. Lines and planes in space. Rectangular coordinate systems. Rigid transformations in space. Rotations and translations of a coordinate system. Distance between two points. Algebraic description of sets in space. Spheres and balls. Vectors. Vector algebra. Parallelogram rule. The dot product. Geometrical significance of the dot product.

Assignment: 1.10 (1, 6, 7, 9, 11, 19, 21), 2.5 (3, 4, 8, 9, 11), 3.8 (2, 4, 5, 9, 11, 15),

Tues: Sections 4-5. The cross product. Geometrical significance of the cross product. Area of a parallelogram. Area of a triangle. Criterion for two vectors being parallel. Triple product. Its geometrical significance. Volume of parallelepiped. Criterion for three vectors being coplanar.

Assignment: 4.6 (2, 3, 4, 11, 16, 19, 26), 5.5 (2, 4, 6, 8, 9, 13)

Wed: Sections 6-7. Algebraic description of lines in space. Vector, parametric, and symmetric equations of a line. Distance between a line and a point. Algebraic description of planes in space. Distance between a plane and a point.

Assignment: 6.5 (1, 3, 7, 9, 11, 21), 7.5 (2, 7, 9, 10, 11, 12, 18, 23).

Thurs: Sections 9-10. Quadric surfaces in space. Classification of quadric surfaces. Paraboloids, hyperboloids, cones, and ellipsoids. Vector functions. Continuous vectors functions and curves in space. Parametric curves and parameterization of a space curve. Graphing parametric curves.

Assignment: 9.6 (1-5, 11-15, 25, 26). 10.4 (1-10).

WEEK TWO (June 1 – June 4): Curves in space and vector functions

Mon: Exam 1 covers Sections 1-9 (one hour).

Sections 11-12. Derivative of a vector function. Its geometric significance. Tangent line to a curve. Smooth curves. Integration of vector functions. Fundamental theorem of calculus for vector functions. Reconstruction of a vector from its derivative. Solving Newton's equations in mechanics.

Assignment: 11.4 (1-3, 6, 7, 11, 19), 12.5 (1-3, 7-10, 12)

Tues: Sections 13-14. Arc length of a curve. Theorem about arc length of a smooth curve. Curvature of a smooth curve. Geometric significance of curvature. Curvature radius. Unit tangent and normal vectors. Osculating plane and circle. Applications to mechanics: tangent and normal accelerations.

Assignment: 13.3(1, 3, 4, 8-9, 16, 17), 14.3 (1-7, 14-17)

Wed: Sections 16-18. Functions of several variables. Range and domain. Graph of a function of two variables as a surface in space. Level sets. Level surfaces of a function of three variables. Limits and continuity. Limit along a curve. Squeeze principle. General strategy to analyze multi-variable limits. Continuity of polynomials. Continuity of composition of continuous functions.

Assignment: 16.4 (4-12, 18-21), 17.3 (1, 2, 8, 9, 23, 24), 18.6 (4-8, 15, 16),

Thurs: Exam 2 covers Sections 10-14 (one hour).

Section 19. Partial derivatives. Geometrical significance of partial derivatives. Partial derivatives

as functions.
Assignment: 19.3 (3-6, 8, 12, 17).

WEEK THREE (June 8 – June 11): Differentiation of functions of several variables

Mon: Sections 20-22. Higher order partial derivatives. Clairaut's theorem. Reconstruction of a function from its partial derivatives. Integrability conditions. Differentiability of a function as the existence of a good linear approximation. Tangent plane approximation for functions of two-variables. Linearization of a function near a point. Continuity of partial derivatives as a sufficient condition for differentiability. Chain rules and implicit differentiation. Implicit function theorem.
Assignment: 20.4 (2-6, 10, 14, 20), 21.6 (1, 5, 7-10, 14, 15, 23), 22.7(2, 6, 7, 11, 25, 26, 31)
Tues: Sections 23-24. Differential. Its geometric significance. Higher order differentials and Taylor polynomials. Accuracy of a linear approximation. Taylor polynomial approximations. Estimation of error of the Taylor approximation for functions with continuous partial derivatives. Directional derivative. Its geometric significance. The gradient and its geometric significance.
Assignment: 23.7 (1-3, 9, 10, 27, 28), 24.5(4-6, 9, 11, 15, 24, 28, 29, 45)
Wed: Sections 25- 26. Extreme values. Critical points and the gradient. Second derivative test for functions of several variables. Extreme values on a set. Extreme value theorem.
Assignment: 25.6(1, 6-10, 13-15), 26.6(1-3, 27-29).
Thurs: Sections 27-29. Extreme values of a function of several variables on smooth curves and surfaces. The Lagrange multiplier method. The volume problem. Lower and upper sums to estimate a volume under a surface. The double integral and its properties.
Assignment: 27.6(3-6, 9, 11), 28.6(2, 10, 11, 15, 16), 29.1(3, 4, 8-10)

WEEK FOUR (June 15 – June 18): Multiple integrals

Mon: Exam 3 covers Sections 15-27 (one hour).
Sections 30-31. Iterated integrals. Fubini's theorem. Evaluation of double integrals over general regions. Vertically and horizontally simple planar regions and their algebraic description. Reversing the order of integration in an iterated integral.
Assignment: 30.3(1-8), 31.6(1-3, 6-13, 28-33)
Tues: Sections 32-33. Reversible transformations in a plane. Change of variables in a plane. Jacobian. Change of variables in double integrals. Example: Double integral in polar coordinates. Symmetry and area preserving transformations in double integrals. The mass problem. Definition of a triple integral. Properties of a triple integral.
Assignment: 32.5(1-3, 9-11, 17-19, 27, 28), 33.5(11-13, 19, 21-23)
Wed: Sections 34-36. Fubini's theorem for triple integrals. Evaluation of triple integral over vertically simple spatial regions. Reversible transformations in space. Change of variables. Jacobian. Change of variables in triple integrals. Example: Triple integrals in spherical and cylindrical coordinates. Volume of an ellipsoid.
Assignment: 34.6(1, 2, 4, 7, 8, 13, 18, 21-23), 35.6(4-6, 9-11, 18-27)
Thurs: Sections 38-39. Line integral along a smooth curve. Surface integral over a smooth

surface. Surface area. Evaluation of the surface integral over a graph of function of two variables.
Assignment: 38.3(1, 3, 5, 14, 18), 39.5(2-4, 6, 7, 9, 28).

WEEK FIVE (June 22 – June 25): Vector fields and vector calculus

Mon: Exam 4 covers Sections 28-39 (one hour).
Sections 41-42. Fluid and gas flows. Velocity of the flows. Vector fields. Graphic representation of a vector field. Flow lines. Line integral of a vector field over a smooth curve. Application: Work done by a force field in moving an object along a smooth curve. Conservative vector fields. Fundamental theorem for line integrals of conservative vector fields. Path-independence of line integrals and the curl of a vector field.
Assignment: 41.6(1-3, 7, 11, 17-19), 42.6(9-12, 17, 18)
Tues: Sections 44-45. Flux of a vector field across a smooth surface. Orientable surfaces. Reduction of the flux integral to a double integral. Stokes' theorem. Induced orientation of a surface by an orientation of its boundary. Reduction of a line integral over a closed path to a flux integral.
Assignment: 44.5 (1-5, 8-11, 15), 45.7 (4-8)
Wed: Sections 43, 46. Two-dimensional version of Stokes'. Green's theorem. Area of a planar region bounded by a smooth closed curve via the line integral over the curve. Flux of a vector field across a smooth orientable closed surface. Divergence (Gauss-Ostrogradsky) theorem. Divergence of a vector field. Geometric significance of the divergence and the curl of vector fields with an example of flow lines of a fluid. Sink and faucet type of sources of a vector field.
Assignment: 43.5(4-10), 46.6 (18-25)
Thurs: Discussion and **Exam 5 covers Sections 41-46 (one hour).**