



**University of International Business and Economics  
International Summer School**

**MAT 230 Multivariable Calculus (Calculus III)**

**Term: October 26<sup>th</sup> – November 20<sup>th</sup>, 2020**

**Instructor: Shen Fan**

**Home Institution: China University of Petroleum**

**Class Hours: Monday through Friday, 120 minutes each day (2,400 minutes in total)**

**Office hours: to be announced**

**Discussion sessions: each Wednesday, time TBD**

**Email: TBD**

**Total Contact Hours: 64 contact hours (45 minutes each, 48 hours in total)**

**Location: WEB**

**Credit: 4 units**

**Course Description:**

The course covers the following concepts: vector algebra, lines, planes, curves, and surfaces in space, functions of several variables, multivariable limits and continuity, partial derivatives and differentiation of functions of several variables, extreme values of functions of several variables and the method of Lagrange multipliers, double and triple integrals, change of variables in multiple integrals, line and surface integrals, and applications of differentiation and multiple integration to vector fields (line and surface (flux) integrals of vector fields, fundamental theorem for line integrals, etc.).

**Course Goals:**

1. Have facility with the basic theory and techniques of integral and differential vector calculus: e.g., the various types of vector products, notions of arc length and curvature, generalizations of the derivative (partial derivatives, directional derivatives, etc.); integrals of multivariable functions, change of variables, vector fields, line integrals, divergence, gradient and curl, integrals of vector fields over surfaces, etc.
2. Have precise knowledge of the definitions, theorems, and derivations from the basic theory of multivariable calculus: e.g., the geometric interpretation of the dot product, various formulae for the arc length, the relationship between gradient and directional derivatives, the change of variable formula, and various generalizations of the fundamental theorem of calculus.
3. Have facility with basic calculational skills: e.g., facility with vectors, evaluation of arc length and curvature, ability to determine tangent planes, facility with the Lagrange multiplier method, ability to calculate double and triple integrals, surface integrals, etc.

4. Have a rudimentary ability to explain mathematical theory using rigorous mathematical reasoning.

**Required Textbook:**

Stewart, James. *Multivariable Calculus*, 6th edition

**Attendance:**

Students are expected to be present at all class meetings and examinations.

**Prerequisites:**

The course is based on Calculus II or its equivalent.

**Grading Policy:**

There will be daily quizzes, three midterms and one final exam in this class. All exams will be closed-book. No notes, calculators, or other electronic devices will be allowed, and having such a device in view during the exam is an academic honesty violation.

<b>A</b>	90 and above	<b>C+</b>	65-69
<b>A-</b>	85-89	<b>C</b>	60-64
<b>B+</b>	80-84	<b>C-</b>	55-59
<b>B</b>	75-79	<b>D</b>	50-54
<b>B-</b>	70-74	<b>F</b>	below 50

It should be noted that in many US colleges **C-** is not a passing grade if the course is required for a major.

**Grading Policy:**

There will be one midterm and a final exam in this class. All exams will be closed-book. No notes, calculators, or other electronic devices will be allowed, and having such a device in view during the exam is an academic honesty violation.

**Grading Scale:**

The course grades will be calculated based on the following percentages:

- Homework: 30%
- Midterm: 30%
- Final Exam: 40%

The final exam will be cumulative. There will be no make-up exams.

**Class Rules:**

All academic work should be done with the high level of honesty and integrity. Academic misconduct of any kind may result in a grade penalty or the assignment of a failing grade.

**Course Schedule: (tentative)**

Day	Topics	Homework
1	Three-dimensional Coordinates.	13.1: 6, 11, 15, 19, 20, 36.
	Vectors.	13.2: 19, 22, 23, 24, 35, 36ab, 39, 43
2	Dot Product.	13.3: 7, 9, 11, 16, 23, 37, 41, 57, *58 (bonus problem)
	Cross product.	13.4: 14, 20, 28, 33, 37, 45, *46 (bonus)
3	Lines and Planes.	13.5: 10, 13, 14, 24, 31, 43, 50, 69 (see example 8)
	Cylinders and Quadric Surfaces.	13.6: 12, 15, 23, 30, 33, 42, 43
4	Vector Valued Functions and Space Curves; Derivatives and Integrals of Vector Functions.	14.1: 5, 15, 19-24, 25, 27, *45
	Derivatives and Integrals of Vector Functions; Arc Length.	14.2: 5, 12, 13, 19, 24, 34, 38, 39; 14.3: 3, 4, 8, 10, 13
5	Arc Length and Curvature.	14.3: 15, 16, 18, 19
6	Arc Length and Curvature.	14.3: 1, 23, 25, 27, 29, 40, 53, *54
	Functions of Several Variables.	15.1: 13, 24, 28, 30, 41, 45, 61, 63
7	Limits and Continuity.	15.2: 1, 9, 10, 11, 15 (tricky?), 20
	Partial Derivatives.	15.2: 6, 13 (squeeze), 17, 19, 33, 37, 38, 41
8	Partial Derivatives.	15.3: 6, 9, 10, 17, 24, 28, 43, 57. Bonus: Let $f(x) = x$ if $x$ is rational; 0 if $x$ is irrational. Show that $f$ is continuous at 0, but discontinuous everywhere else.
	Tangent Planes Differentiability and Chain Rule.	15.4: 5, 6, 11, 17, 18, 19, 25, 26, *35, 42
9	Chain Rule.	15.5: 2, 5, 6, 7, 11, 21, 27 (read Equation 6), 45 (used in physics), *55
	Gradients, Directional Derivatives.	15.6: 7, 10, 13, 15, 16, 21, 23, 34
10	Max-Min Problems.	15.6: 27, 32, *37, 39, 42, 47, 50; 15.7: 5, 6, 7, 13, 14, 19, 43, 51
11	Lagrange Multipliers.	15.8: 5, 6, 7, 8
	Double integrals intro.	15.8: 12, 14, 15, 16, 41, *45, *46
12	Iterated Integrals.	16.2: 1, 7, 8, 16, 17, 18, 24
	Double Integrals	16.3: 2, 5, 6, 9, 11, 14, 17, 20
13	Double Integrals Continued.	16.3: 21, 24, 25, 27, 41, 42, 43, 47, 53
	Double Integrals in Polar Coordinates.	16.4: 5, 6, 7, 11, 12 (tricky?), 16, 22, 26, 30, *36

14	Triple Integrals.	16.6: 3, 7, 9, 11, 14, 21, 22 [can use polar coordinates for 21, 22], 28, 36.
	Triple Integrals in Cylindrical Coordinates.	16.7: 9, 12, 15, 19, 21, 22, *26
15	Triple Integrals in Spherical Coordinates.	16.8: 9, 13, 15, 21, 24 (int. by parts), 26, 28, 39
16	Change of Variable Formula.	16.9: 1, 2, 3, 6, 7, 9, 10, 11
	Vector Fields.	16.9: 12, 14, 15, 17, 19, 21, 22; 17.1: 6, 7, 21, 23, 29, 34, 35
17	Line Integrals.	17.2: 3, 5, 7, 11, 12, 15; 17.2: 18, 19, 20, 21, 22
	Fundamental Theorem of Calculus for Line Integrals.	17.3: 13, 15, 16, 20, 21, 27, 29, 31
18	Green's Theorem.	17.4: 1, 3 (somewhat of a pain), 5, 7, 8 (switch to polar at some point), 9, *13 (not hard, just a matter of understanding the notation)
	Div, Grad, Curl.	17.5: 1, 5, 6, 7, 13, 15, 19
19	Parametric Surfaces. Surface Integrals.	17.6: 13-15, 17, 21, 23, 24; 17.6: 33, 34, 37, 41, 42, 47; 17.7: 5, 7, 9, 15
	Surface Integrals, continued.	17.7: 19, 21, 22, 23, 25
20	Stokes's Theorem.	17.8: 1, 3, 6, 8, 15, 16

### General Comments:

- ✧ In order to train your minds in mathematical thinking, much of lecture will consist of the "Socratic Method" of questioning. Even if silently, do try to puzzle out the answers. An analogy: the lecture should be a "mental exercise" class. Knowledge cannot be given: it must be stolen. Engage your mind.
- ✧ Many problems will not be solvable at first (or second) viewing. Be patient: clarify any unknown concepts, try to reduce the problem, brainstorm to unearth possibly relevant concepts, and follow your intuition. It may help to, after a period of hard work, put the problem away. Do not worry if you cannot do every problem: what is important is that you try.
- ✧ If you find yourself getting lost in the material, come to office hours immediately! It is much easier to lead a student's mind individually rather than in a group.
- ✧ Note that we may not dot every i or cross every t in class: you are responsible for reading the text. In particular, it is to your advantage to read the material before coming to class; in this way the student becomes an active participant rather than a passive recipient.
- ✧ Basic etiquette should be maintained. For example: to give your classmates time to think, please do not blurt out answers unless called upon (or overcome with excitement); please do not walk out of class without prior explanation, etc.
- ✧ Using a cell phone in class is not permitted; students using phones will be penalized 5 percentage points on the next exam.



- ✧ Tests will check your understanding of the lectures as well as cover homework-type problems; it will benefit you to check after each lecture to see if you've understood the line of the arguments. Precise knowledge of the theory is vital!