

# University of International Business and Economics International Summer School

## MAT 230 Multivariable Calculus (Calculus III)

Term: September 20<sup>th</sup> – December 10<sup>th</sup>, 2021 Instructor: Shen Fan Home Institution: China University of Petroleum Email: fans@cup.edu.cn Class Hours: 2 days per week, 120 minutes each teaching day (2,400 minutes in total) Office hours: to be announced Discussion sessions: each Wednesday, time TBD

### Total Contact Hours: 64 contact hours (45 minutes each, 48 hours in total) Credit: 4 units

#### **Course Description:**

The course covers the following concepts: vector algebra, lines, planes, curves, and surfaces in space, functions of several variables, multivariable limits and continuity, partial derivatives and differentiation of functions of several variables, extreme values of functions of several variables and the method of Lagrange multipliers, double and triple integrals, change of variables in multiple integrals, line and surface integrals, and applications of differentiation and multiple integration to vector fields (line and surface (flux) integrals of vector fields, fundamental theorem for line integrals, etc.).

#### **Course Goals:**

- 1. Have facility with the basic theory and techniques of integral and differential vector calculus: e.g., the various types of vector products, notions of arc length and curvature, generalizations of the derivative (partial derivatives, directional directives, etc.); integrals of multivariable functions, change of variables, vector fields, line integrals, divergence, gradient and curl, integrals of vector fields over surfaces, etc.
- 2. Have precise knowledge of the definitions, theorems, and derivations from the basic theory of multivariable calculus: e.g., the geometric interpretation of the dot product, various formulae for the arc length, the relationship between gradient and directional derivatives, the change of variable formula, and various generalizations of the fundamental theorem of calculus.
- 3. Have facility with basic calculational skills: e.g., facility with vectors, evaluation of arc length and curvature, ability to determine tangent planes, facility with the Lagrange multiplier method, ability to calculate double and triple integrals, surface integrals, etc.
- 4. Have a rudimentary ability to explain mathematical theory using rigorous mathematical reasoning.



#### **Required Textbook:**

Stewart, James. Multivariable Calculus, 7th edition

#### Attendance:

Summer school is very intense and to be successful, students need to attend <u>every class</u>. Occasionally, due to illness or other unavoidable circumstance, a student may need to miss a class. A medical certificate is required to be excused. Any absence may impact on the student's grade. Arriving late or leaving early will count as a partial absence. If a student is missing less than a point for a better grade, the better grade will be given, provided the student had no unexcused absences during the course.

#### Prerequisites:

The course is based on Calculus 1 and 2 (or their equivalents). Students are expected to know basic concepts of calculus for functions of a single real variable. Good technical skills in differentiation and integration are necessary. Particular topics of Calculus 2, such as numerical series, power series, planar curves, are not mandatory for the course. However, a basic knowledge of these topics will be very helpful as the course contains higher dimensional versions of them.

#### Grading Policy:

There will be one midterm and a final exam in this class. All exams will be closed-book. No notes, calculators, or other electronic devices will be allowed.

The course grades will be calculated based on the following percentages:

- Homework: 30%
- Midterm: 30%
- Final Exam: 40%

The final exam will be cumulative. There will be no make-up exams.

#### Grading Scale:

Assignments and examinations will be graded according to the following grade scale:

| Α  | 90-100 | C+ | 72-74    |
|----|--------|----|----------|
| A- | 85-89  | С  | 68-71    |
| B+ | 82-84  | C- | 64-67    |
| В  | 78-81  | D  | 60-63    |
| В- | 75-77  | F  | below 60 |

It should be noted that in many US colleges **C**- is not a passing grade if the course is required for a major.



## **Class Rules:**

All academic work should be done with the high level of honesty and integrity. Academic misconduct of any kind may result in a grade penalty or the assignment of a failing grade.

### **Course Schedule: (tentative)**

| Week | Topics                                                                                      |
|------|---------------------------------------------------------------------------------------------|
| 1 -  | Three-dimensional Coordinates.                                                              |
|      | Vectors.                                                                                    |
|      | Dot Product.                                                                                |
|      | Cross product.                                                                              |
| 2    | Lines and Planes.                                                                           |
|      | Cylinders and Quadric Surfaces.                                                             |
|      | Vector Valued Functions and Space Curves; Derivatives and Integrals of Vector<br>Functions. |
|      | Derivatives and Integrals of Vector Functions; Arc Length.                                  |
| 3    | HOLIDAY                                                                                     |
| 4    | Arc Length and Curvature.                                                                   |
|      | Arc Length and Curvature.                                                                   |
|      | Functions of Several Variables.                                                             |
| 5 -  | Limits and Continuity.                                                                      |
|      | Partial Derivatives.                                                                        |
|      | Partial Derivatives.                                                                        |
|      | Tangent Planes Differentiability and Chain Rule.                                            |
| 6    | Chain Rule.                                                                                 |
|      | Gradients, Directional Derivatives.                                                         |
|      | Max-Min Problems.                                                                           |
| 7    | Lagrange Multipliers.                                                                       |
|      | Double integrals intro.                                                                     |
|      | Iterated Integrals.                                                                         |



|    | Double Integrals                                    |
|----|-----------------------------------------------------|
|    | Double Integrals Continued.                         |
| 8  | Double Integrals in Polar Coordinates.              |
| 8  | Triple Integrals.                                   |
|    | Triple Integrals in Cylindrical Coordinates.        |
|    | Triple Integrals in Spherical Coordinates.          |
| 9  | Change of Variable Formula.                         |
|    | Vector Fields.                                      |
|    | Line Integrals.                                     |
| 10 | Fundamental Theorem of Calculus for Line Integrals. |
| 10 | Green's Theorem.                                    |
|    | Div, Grad, Curl.                                    |
| 11 | Parametric Surfaces. Surface Integrals.             |
|    | Surface Integrals, continued.                       |
| 12 | Stokes's Theorem.                                   |